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Reg. No. : .....

Name : .....

Second Semester B.Sc. Degree Examination, May 2020

First Degree Programme under CBCSS

Mathematics

Foundation Course – II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. Define conjunction.
2. What is a biconditional statement?
3. Define contradiction.
4. Give an example of a tautology.
5. Prove that empty set is a subset of any set A.
6. Find the curve represented by the parametric equation $x = t^2, y = t^3$.
7. Write the arc length formula for parametric curves.
8. Define a parabola.
9. Find the distance d between the points $(2, 3, -1)$ and $(4, -1, 3)$.
10. Define orthogonal projection of v on e.

SECTION – II

Answer **any eight** questions from among the questions 11 to 22. These questions carry **2** marks each.

11. Find the antecedent and consequent in the following statement. "You can work here only if you have a college degree."

12. Determine the truth value of the statement $\forall x \exists y \ni x + y = 3$. Justify.

13. Prove that $|x| \geq 0, \forall x$.

14. Prove that $A \cup (U / A) = U$.

15. Define injuctive and surjective function. Give example for each.

16. If $f(x) = \sin x$ is a function defined on $\boldsymbol{R}$. Find $f([0, \pi])$ and $f([0, 8\pi])$.

17. Sketch the curve whose parametric equation is $x = \cos t, y = \sin t$.

18. Find parametric equations for a circle of radius 2, centered at (3, 5).

19. Find the rectangular coordinates of the point P whose polar coordinates are $(r, \theta) = (6, 2\pi / 3)$.

20. Find the unit vector that has the same direction as $v = 2i + 2j - k$.

21. Find the angle between the vector $u = i - 2j + 2k$ and $v = -3i + 6j + 2k$.

22. Let $u = \langle 1, 2, -2 \rangle$ and $v = \langle 3, 0, 1 \rangle$. Find

    (a) $u \times v$

    (b) $v \times u$.

SECTION – III

Answer **any six** questions from among the questions 23 to 31. These questions carry **4** marks **each.**

23. Prove that "If 7m is an odd number then m is an odd number".

24. Which of the following statements are true? Justify

    (a) If $m^2 > 0$ then m 0

    (b) If $m > 0$ then $m^2 > 0$.

25. Prove that $[(p \wedge \sim q) \Rightarrow c] \Leftrightarrow (p \Rightarrow q)$ is a tautology.

26. Find polar coordinates of the point P whose rectangular coordinates are $(-2, -2\sqrt{3})$.

27. Sketch the graph of $r = \theta(\theta \geq 0)$ in polar coordinates by plotting points.

28. State symmetry test.

29. State any four rules of vector arithmetic.

30. Find the direction cosines of the vector $v = 2i - 4j + 4k$, and approximate the direction angles to the nearest degree.

31. Find the angle between a diagonal of a cube and one of its edges.

SECTION – IV

Answer **any two** questions from among the questions 32 to 35. These questions carry **15** marks each.

32. Explain in detail any five proof techniques.

33. (a) Prove that the subset of a countable set is countable.

    (b) Prove that the set of all real numbers is uncountable.

34. In a disastrous first flight, an experimental paper airplane follows the trajectory of the particle having parametric equation $x = t - 3\sin t$, $y = 4 - 3\cos t (t \geq 0)$; but crashes into a wall at time $t = 10$.

(a) At what times was the airplane flying horizontally?

(b) At what times was it flying vertically?

35. (a) Find the distance D between the point $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$.

(b) Describe the surface $4x^2 + 4y^2 + z^2 + 8y - 4z = -4$.

(c) Find equations of the paraboloid $z = x^2 + y^2$ in cylindrical and spherical coordinates.
